MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

TMA 1111 – MATHEMATICAL TECHNIQUES

(All sections / Groups)

14 October 2019
2.30P.M. – 4.30P.M.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of **4 pages** only excluding the cover page with **5 Questions** and an **Appendix**.
2. Attempt **ALL FIVE (5)** questions.
3. Please write your answers in the Answer Booklet provided, and **start each question on a new page.**
4. Show all steps.
5. Formulas are provided and attached in Appendix.

Question 1

a. Consider the points $P(1, 4, 4)$, $Q(1, 1, 1)$ and $R(-2, 3, 1)$.

i. Find the vectors $u = \overrightarrow{PQ}$ and $v = \overrightarrow{PR}$. [1 mark]

ii. Find the cross product of u and v, that is $u \times v$. [2 marks]

iii. Determine the equation of the plane passing through the point P, Q and R. [2 marks]

b. Given that $A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$.

i. Find the characteristics equation. Then, find the eigenvalues. [1.5 marks]

ii. Find the eigenvectors. [3.5 marks]

Question 2

a. Given the complex numbers $z = 10+10i$ and $w = 2-7i$,

i. find zw. [1.5 marks]

ii. find $\frac{1}{w}$. [2 marks]

iii. express z in polar form and exponential form. [3 marks]

b. Calculate $\left\{2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)\right\}^4$. [1.5 marks]

c. Simplify $\frac{(\cos 2\theta + i\sin 2\theta)^3(\cos\theta + i\sin\theta)^4}{(\cos 0.5\theta + i\sin 0.5\theta)^2}$. [2 marks]

Question 3

a. Given $f(x) = \begin{cases} \sqrt{x^3 - x} & , x \geq 2 \\ 3x & , x < 2 \end{cases}$.

Compute $\lim_{x \to 2^+} f(x)$ and $\lim_{x \to 2^-} f(x)$. Hence, determine whether $\lim_{x \to 2} f(x)$ exists by applying the suitable limit theorem(s) or rule(s). [2.5 marks]

b. Find the derivative of the following functions.

i. $y = (2 - x^3 + 5x)(\sin x + 9)$ by using product rule or some other method. [2 marks]

ii. $y = \left(2.5x - 3x^2 + e^x\right)^3$ by using chain rule or some other method. [1.5 marks]

c. Integrate

i. $\int_{-1}^{0} (\cos x + x^5 - 6)\ dx$. [2 marks]

ii. $\int 3x^4 \sin x\ dx$ by using integration by part. [2 marks]

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Question 4

a. Solve the following first order differential equation using separable method.

$$\frac{dy}{dx} = \frac{4x^3}{y^2 + 6y}$$ [2 marks]

b. Given the first order differential equation: $9y'+9y = 9x$, rewrite it into the form of $y'+p(x)y = r(x)$.

Then, find the integrating factor, $u(x) = e^{\int p(x)dx}$. [1.5 marks]

c. It is known that the second order differential equation is $ay''+by'+cy = 0$.
Given that the roots of the characteristic equation are $\lambda_1 = -5$ and $\lambda_2 = -5$, find the values of a, b and c. [2.5 marks]

d. Given that the non-homogeneous differential equation, $y''-5y'+4y = 5x$, has $y_h = c_1e^x + c_2e^{4x}$,

i. find the particular solution, y_p, using undetermined coefficients method. [3.5 marks]

ii. form the general solution, $y = y_h + y_p$. [0.5 mark]

Question 5

A lecturer wants to determine whether the midterm test score had any effect on the final exam score of his subject. He randomly selected 10 students and the following data was recorded.

Midterm Test Score	15	19	18	12	20	16	9	11	8	9
Final Exam Score	22	47	44	37	50	31	24	36	24	20

a. Find the $\sum x$, $\sum y$, $\sum x^2$, $\sum y^2$ and $\sum xy$. [2.5 marks]

b. Find regression line to fit the data. [4 marks]

c. Using the regression line from part (b), what will a student score for his final exam if he scored 10 on the midterm test? [1 mark]

d. Calculate the correlation coefficient, r. What does this value indicate? [2.5 marks]

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APPENDIX

1. Equation of plane passing through (x_0, y_0, z_0) with normal vector $<a, b, c>$:
$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

2. Characteristics equation: $|\lambda I - A| = 0$

3. For eigenvectors: $(\lambda \mathrm{I} - A)X = 0$

4. Complex Numbers

$r = \sqrt{x^2+y^2}$

If x is positive, $\theta = \tan^{-1}\frac{y}{x}$

If x is negative, $\theta = \tan^{-1}\frac{y}{x} + \pi$

Polar form: $z = r(\cos\theta + i\sin\theta)$

Exponential form: $z = re^{i\theta}$

De Moivre's Formula: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

5. General solution of the homogeneous linear second order differential equation

For Distinct real roots: $y = c_1e^{\lambda_1 x} + c_2e^{\lambda_2 x}$

For Repeated roots: $y = c_1e^{\lambda_1 x} + c_2xe^{\lambda_2 x}$

For Complex conjugate roots: $y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

6. Derivative Formulas

$\frac{d}{dx}e^x = e^x$
$\frac{d}{dx}\sin x = \cos x$
Power Rule: $\frac{d}{dx}x^n = nx^{n-1}$ if $f(x) = x^n$ with $n \in R$
Product Rule: $\frac{d}{dx}(f \cdot g) = f\frac{dg}{dx} + g\frac{df}{dx}$
Quotient Rule: $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2}$ with $g(x) \neq 0$
Chain Rule: If $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

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7. Integration Formulas

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, for $n \neq -1$, n rational
$\int e^x dx = e^x + C$
$\int \sin kx dx = -\frac{\cos kx}{k} + C$
$\int \cos kx dx = \frac{\sin kx}{k} + C$
Integration by Substitution: If $u = g(x)$ and $du = g'(x)dx$, then $\int f(u)du = \int f(g(x))\frac{d}{dx}g(x)dx$
Integration by Part: $\int udv = uv - \int vdu$

8. Regression

$$\bar{x} = \frac{\sum x}{n}$$

$$S_{xy} = \sum xy - \frac{\left(\sum x \sum y\right)}{n}, \qquad S_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n}, \qquad S_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n}$$

$$\beta_1 = \frac{S_{xy}}{S_{xx}}, \qquad \beta_0 = \bar{y} - \beta_1 \bar{x}$$

Least square regression line: $y = \beta_0 + \beta_1 x$

Correlation coefficient : $r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

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